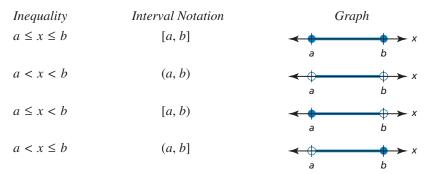## Using Interval Notation

Another way to represent solutions of inequalities is to use *interval notation*.
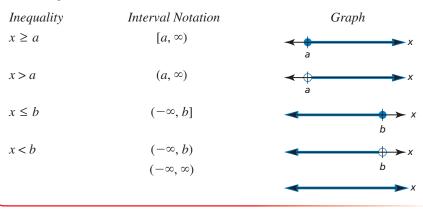
KEY IDEA

Interval Notation

Let a and b be two real numbers such that $a < b$. Then a and b are the **endpoints** of four different **bounded intervals** on the real number line, as shown below. A bracket indicates that the endpoint is included in the interval, and a parenthesis indicates that the endpoint is not included in the interval.

Inequality	Interval Notation	Graph
$a \leq x \leq b$	$[a, b]$	
$a < x < b$	(a, b)	
$a \leq x < b$	$[a, b)$	
$a < x \leq b$	$(a, b]$	

The length of any bounded interval is the distance between its endpoints: $b - a$. Any bounded interval has a *finite* length. Each interval shown below is an *unbounded* interval. An **unbounded interval** is an interval that does not have a finite length.
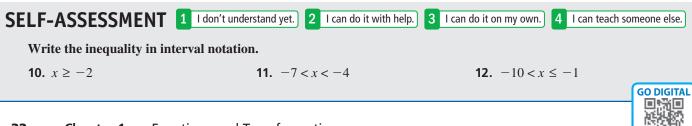
Inequality	Interval Notation	Graph
$x \geq a$	$[a, \infty)$	
$x > a$	(a, ∞)	
$x \leq b$	$(-\infty, b]$	
$x < b$	$(-\infty, b)$	
	$(-\infty, \infty)$	

STUDY TIP

The symbols ∞ (*infinity*) and $-\infty$ (*negative infinity*) are used to represent the unboundedness of intervals such as $[7, \infty)$ and $(-\infty, 7]$. Because these symbols do not represent real numbers, they are always enclosed by a parenthesis.

EXAMPLE 3   **Writing Interval Notation**   ▶ WATCH

Write each inequality in interval notation.

a. $-2 \leq x \leq 3$ **b.** $x > -1$

SOLUTION

a. The inequality $-2 \leq x \leq 3$ is the bounded interval $[-2, 3]$.

b. The inequality $x > -1$ is the unbounded interval $(-1, \infty)$.

SELF-ASSESSMENT
1 I don't understand yet.
2 I can do it with help.
3 I can do it on my own.
4 I can teach someone else.

Write the inequality in interval notation.

10. $x \geq -2$ **11.** $-7 < x < -4$ **12.** $-10 < x \leq -1$

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1.4 Practice WITH **CalcChat**® AND **CalcView**®

In Exercises 1–16, solve the inequality. Graph the solution, if possible. *(See Examples 1 and 2.)*

1. $|x| < 3$

2. $|y| \geq 4.5$

▶ **3.** $|d + 9| > 3$

4. $|h - 5| \leq 10$

5. $|2s - 7| \geq -1$

6. $|4c + 5| > 7$

7. $|5p + 2| < -4$

8. $|9 - 4n| < 5$

9. $|6t - 7| - 8 \geq 3$

10. $|3j - 1| + 6 < 0$

11. $3|14 - m| > 18$

12. $-4|6b - 8| \leq 12$

▶ **13.** $2|3w + 8| - 12.5 \leq -4.5$

14. $-3|2 - 4u| + 5.5 < -4.1$

15. $6\left|-\frac{1}{4}f + 3\right| + 7 > 7$ **16.** $\frac{2}{3}|4v + 6| - 2 \leq 10$
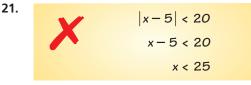
In Exercises 17 and 18, write the sentence as an absolute value inequality. Then solve the inequality.

17. A number is less than 6 units from 0.

18. Twice a number is no less than 10 units from -1.

19. MODELING REAL LIFE The rules for an essay contest require that entries have 500 words with an absolute deviation of at most 30 words. Write and solve an absolute value inequality that represents the acceptable numbers of words.

20. MODELING REAL LIFE The normal body temperature of a camel is 37°C. This temperature varies by up to 3°C throughout the day. Write and solve an absolute value inequality that represents the range of normal body temperatures (in degrees Celsius) of a camel throughout the day.

ERROR ANALYSIS **In Exercises 21 and 22, describe and correct the error in solving the absolute value inequality.**

21.

✗
$|x - 5| < 20$
$x - 5 < 20$
$x < 25$

22.

✗
$|x + 4| > 13$
$x + 4 > -13 \quad \text{and} \quad x + 4 < 13$
$x > -17 \quad \text{and} \qquad x < 9$
$-17 < x < 9$

In Exercises 23–28, write the inequality in interval notation. *(See Example 3.)*

▶ **23.** $3 < x < 9$

24. $-5 < x \leq 20$

25. $x \geq -13$

26. $x \leq 58$

27. $x < 19$

28. $x > 71$

In Exercises 29 and 30, solve the inequality. Then write the solution in interval notation.

29. $|x - 7| < 12$

30. $|x + 2| \leq 5$

In Exercises 31–34, write an inequality that represents the situation. Then write the inequality in interval notation.

31. An animal shelter needs at least 110 pounds of dog food for a month.

32. You must release fish that weigh less than 1.2 pounds.

33. Your hourly wage of $15 differs from the average by less than or equal to $5.

34. An essay contest requires that entries be within 50 words of 1000 words long.

35. MODELING REAL LIFE The elevation relative to sea level in the United States ranges from -282 feet in Death Valley, California, to 20,320 feet on Denali in Alaska. Write the range of elevations in interval notation and in set-builder notation.

36. MODELING REAL LIFE The main floor of an auditorium ranges from 6 feet below the stage to 8 feet above the stage. The floor of the balcony ranges from 26 to 37 feet above the stage. Write the range of the floor levels relative to the stage in interval notation and in set-builder notation.

37. MODELING REAL LIFE A manufacturer throws out gaskets with weights that have an absolute deviation of more than 0.06 pound from the mean weight of the batch. The weights (in pounds) of the gaskets in a batch are 0.58, 0.63, 0.65, 0.53, and 0.61. Which gaskets should be thrown out? Use an absolute value inequality to justify your answer. *(See Example 4.)*

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